Off-the-wall 6: Two students get into an argument about air friction and how it affects the motion of a free falling body. One student thinks it is proportional to the body's velocity v, the other think it's proportional to the velocity squared.

a.) What would you have to do to write out a differential equation that would characterize the motion of a body in free fall?

--for the student who thought the force was proportional to the velocity, summing the forces via Newton's Second Law would produce the equation $kv - mg = - m (dv/dt)$;
--for the student who thought the force was proportional to the velocity squared, summing the forces via Newton's Second Law would produce the equation $kv^2 - mg = - m (dv/dt)$;

b.) How would you determine the terminal velocity for that body?

--the terminal velocity occurs when the upward and downward forces become equal, which is to say, when dv/dt goes to zero;
--using the relationship quoted above with $dv/dt = 0$ will allow one to determine the terminal velocity.

c.) Assume you were given twenty coffee filters, a meter stick, a stop watch, a balance, a motion detector and a cell phone with video capabilities. Design an experiment that would allow you to deduce which of the two situations is the reality. Explain the set-up, explain what data you would take and explain how the data would allow you make your deduction.

--actually trying to measure the velocity as a function of time for a single falling object is not easily done;
--as an alternative:
  --the force of air-friction at terminal velocity must equal the weight of the object,
  --the terminal velocity for the "force proportional to velocity" camp is "$kv(terminal) = mg$", which suggests that the mass and terminal velocity are proportional to one another;
  --so take five coffee filters, mass them, then let them drop from 3 meters up over a motion detector—if they hit terminal velocity (which they should), the "velocity versus time" graph from the detector will go constant;
  --(as an alternative to the previous step, set up a meter stick along the flight path and use an iPhone to take a video of the free fall; the terminal velocity should be calculable using distance traveled and time of flight—you can get the latter by counting the number of frames the motion required on the video to go the distance);
  --do this for 5 filters, 10 filters, 15 filters and 20 filters;
  --graph "mg vs v(terminal)";
  --if the graph is linear, air friction will be proportional to v;
  --if the graph is not linear, air friction will not be proportional to v and must therefore be proportional to v^2 (in fact, the graph should look parabolic)

d.) (Don't take a lot of time on this one—it's a weird one.) A third student inserts herself into the conversation right at the beginning and says she thinks the correct function can't be deduce by discussion but might be deduced another way. Her suggestion is to assume an air-friction force magnitude of $F = kv^N$. With that, she maintained that using a body whose mass could be varied (those coffee filters) and finding its terminal velocity for a number of different masses would, with some clever manipulation, allow her to deduce both k and N. She points out that at terminal velocity, the force F exactly off-sets gravity, so it equals mg at that point, and she says that graphing the *natural log of F* versus the *natural log of v* would do the trick. She is then called away before explaining more. Kindly fill in the blanks. Explain how her procedure would help?

--this is just mathematical trickery;
--taking the natural log of both sides of the force question yields:

$$\ln (F) = \ln (k) + N \ln (v)$$

--think about the classic relationship for a straight line: y = mx + b, where "m" is the slope of the graph and "b" is they y-intercept;
--if we graph "ln (F)" (remembering that F will equal "mg" at terminal velocity) along the y-axis and "ln (v)" along the x-axis, then according to that classical relationship "N" must be the slope of the graph and "ln (k)" the y-intercept;
--as I said, this is a tricky problem, and more of a "how clever can you be with the math" kind of thing than anything else . . .